Cavity-assisted fast generation of entangled photon pairs through the biexciton-exciton cascade

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We present a scheme for the fast cavity-assisted generation of entangled photon pairs from a single quantum dot coupled to a planar photonic crystal that support two orthogonally polarized cavity modes. We discuss "within generation" and "across generation" of entangled photons when both the biexciton to exciton, and the exciton to ground-state transitions, are coupled through cavity modes. In the across generation, the photon entanglement is restored through a time delay between the photons. The concurrence, which is a measure of the entanglement between two photons, is greater than 0.7 and 0.8 using experimentally achievable parameters in across generation and within generation, respectively. We also show that the entanglement can be distilled in both cases using a simple spectral filter.

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I. INTRODUCTION

Entangled photons are an essential resource for various quantum information processing protocols,[1,2] such as quantum cryptography[3] and quantum teleportation.[4] The entangled photons employed in most experiments to date have been generated using parametric down conversion.[5,6] However, recent developments of scalable quantum systems[7] require a scalable "on demand" source of entangled photon pairs. With regard to suitable material systems for on demand photon sources, there has been considerable progress for developing entangled photon sources using single quantum dots (QDs).[8–13] In semiconductor QDs, entangled photons can be generated in a biexciton-exciton cascade decay. However, the entanglement between the generated photons is limited by inherent cylindrical asymmetries and various dephasing processes.[14–16] The cylindrical asymmetries produce fine structure splitting (FSS) in the exciton states;[17] as a result, the emitted x-polarized and y-polarized photon pairs become distinguishable in frequencies, and the entanglement between the photons is largely destroyed. Several methods have been employed to minimize the detrimental effects of FSS on the generated photons, for example, by spectrally filtering the indistinguishable photon pairs,[8] by applying external fields to suppress the FSS,[9,10] by thermal annealing the QDs,[11] by selecting QDs with smaller FSS,[12] and by using temporal gates.[13] In all of these approaches, the photons of different polarizations, generated within the same generations, are forced to match in their frequencies.

An interesting alternate approach, insensitive to FSS, has been proposed recently, which suppressing the binding energy of the biexciton.[18–22] For a zero binding energy of the biexciton, photons of different polarizations match in energy in "across generations" (see Fig. 1). Because of the different ordering in the emission for x-polarized and y-polarized photon pairs, the photons are distinguishable temporarily and remain unentangled, but the entanglement can be restored using a time delay between photons of different generations.

The effects of dephasing in the generated entangled state of photons can be minimized significantly by enhancing the emission rates of the photons through the Purcell effect in a system comprised of a QD coupled with a microcavity. Several experiments have also demonstrated single QD strong coupling to semiconductor cavities.[23–25] Recently, Johne *et al.*[26] proposed a cavity-QED (quantum electrodynamics) scheme for generating entangled photons in the strong coupling regime. In their scheme, excitons are strongly coupled with cavity modes and form degenerate polariton states.[27] A formal theory of this scheme, including exciton and biexciton broadenings, has been reported by us.[28] However, one drawback of the proposed method is that because of the large binding energy, the biexciton remains uncoupled with cavity modes and thus the first generation of photons has a significantly long life time, which manifests in a reduced collection efficiency and an increased sensitivity to dephasing processes. Although four-mode cavity coupling was briefly discussed in Ref. 29, the authors concluded that such a scheme was not practical.

In this paper, we propose a scheme for the fast generation of entangled photons from a single QD, by manipulating the binding energy of the biexciton such that both biexciton to excitons and excitons to ground-state transitions are coupled with two cavity modes of orthogonal polarization. Because of the Coulomb interaction, generally the biexciton binding energy has negative values, however by changing the confinement size[30] or by changing the strain,[11] it has been shown earlier that biexciton binding energy can be tuned to zero or

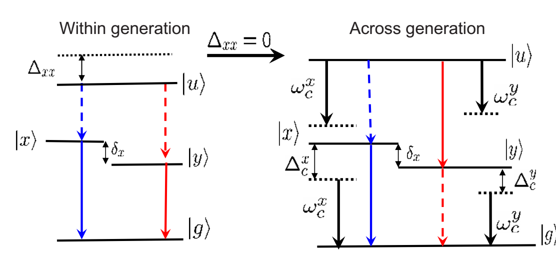

FIG. 1. (Color online) Schematic of the resulting energy level diagram for cavity-QED assisted generation of entangled photons in the biexciton-exciton cascade decay after manipulating the binding energy of the biexciton ($\Delta_{xx}\rightarrow 0$). The biexciton state $|u\rangle$ decays to the ground state $|g\rangle$ via intermediate exciton state $|x\rangle$ or $|y\rangle$, creating an x-polarized or y-polarized photons in the cavity modes. The x-polarized and y-polarized cavity modes are coupled with the $|u\rangle\rightarrow|x\rangle$, $|x\rangle\rightarrow|g\rangle$ and $|u\rangle\rightarrow|y\rangle$, $|y\rangle\rightarrow|g\rangle$ transitions, respectively.

even to a positive value. Very recently, manipulation of the binding energies of the biexciton has been reported by applying lateral electric fields.[18,19] Further in a recent work, construction of an electrode for applying lateral electric field in the vicinity of a QD within a photonic crystal cavity has also been reported.[31] Therefore, it is now possible to manipulate the binding energy of biexcitons within the photonic crystal cavities. Although we are primarily interested in the QD system here, our results are equally applicable for equivalent atomic systems[32] placed inside optical cavities. In the proposed fast generation schemes introduced below, we discuss both "across generation" and "within generation" of entangled photon pairs.

This paper is organized as follows. In Sec. II, we present a formal theory of a single QD coupled to a planar photonic crystal cavity. The cavity-assisted *across generation* of entangled photons is discussed in Sec. III. In Sec. IV, the cavity-assisted *within generation* of entangled photons is presented. In Sec. V, we present our conclusions.

II. THEORY

We consider a QD embedded in a photonic crystal cavity having two orthogonal polarization modes of frequencies ω_c^x and ω_c^y, which can be realized and tuned experimentally using electron-beam lithography and, for example, AFM oxidization techniques.[33] The exciton states, $|x\rangle$ and $|y\rangle$, have a FSS, δ_x. The cavity modes are coupled with the biexciton to exciton, and the exciton to ground-state transitions, by manipulating the biexciton binding energy.[11,18,19] The schematic arrangement of the system is shown in Fig. 1. Of course, a real QD will have other exciton and biexcitonic states as well;[34] however, a representative model, in the spectral vicinity of interest, is one that includes only the target biexciton and the two lower lying exciton states, resulting in the well known biexciton-exciton cascade scheme. In the presence of a cavity, this model is even better, as the other levels remains spectrally uncoupled. We remark, further, that most of the experimental biexciton-exciton systems in QDs have been successfully explained using this simplified model.

The Hamiltonian for the system with a QD coupled with two-modes in a photonic crystal cavity, in the interaction picture, can be written as

$$H_I(t) = \hbar\Bigg[g_1^x|x\rangle\langle g|\hat{a}_c^x e^{i\Delta_c^x t} + g_2^x|u\rangle\langle x|\hat{a}_c^x e^{i(\omega_{ux}-\omega_c^x)t} + g_1^y|y\rangle \times\langle g|\hat{a}_c^y e^{i\Delta_c^y t} + g_2^y|u\rangle\langle y|\hat{a}_c^y e^{i(\omega_{uy}-\omega_c^x)t} + \sum_{m\neq c}\Omega_{xm}\hat{a}_c^{x\dagger}\hat{a}_m^x e^{i(\omega_c^x-\omega_m)t} + \sum_{m\neq c}\Omega_{ym}\hat{a}_c^{y\dagger}\hat{a}_m^y e^{i(\omega_c^y-\omega_m)t}\Bigg] + \text{H.c.}, \tag{1}$$

where $\omega_{ux}=\omega_u-\omega_x$, $\omega_{uy}=\omega_u-\omega_y$, $\Delta_c^x=\omega_x-\omega_c^x$, $\Delta_c^y=\omega_y-\omega_c^y$, and $\hat{a}_j^i$ are the field operators with $\hat{a}_c^x$ and $\hat{a}_c^y$ the cavity mode operators. Here, Ω_{xm}, and Ω_{ym} represent the couplings to the environment from the x-polarized and y-polarized cavity modes; g_j^i are the coupling strengths between the exciton/biexciton and cavity modes; ω_m are the frequencies of the photons emitted from the cavity modes, and ω_u, ω_x, and ω_y are the frequency of the biexciton and excitons, respectively. We consider a system that is optically pumped in such a way as to have an initially excited biexciton, with no photons inside the cavity, thus, the state of the system at any time t can be written as follows:

$$|\psi(t)\rangle = c_1(t)|u,0,0\rangle|0\rangle_x|0\rangle_y + c_2^x(t)|x,1,0\rangle|0\rangle_x|0\rangle_y + c_2^y(t)|y,0,1\rangle|0\rangle_x|0\rangle_y + c_3^x(t)|g,2,0\rangle|0\rangle_x|0\rangle_y + c_3^y(t)|g,0,2\rangle|0\rangle_x|0\rangle_y + \sum_m c_{4m}^x(t)|x,0,0\rangle|1_m\rangle_x|0\rangle_y + \sum_m c_{4m}^y(t)|y,0,0\rangle|0\rangle_x|1_m\rangle_y + \sum_m c_{5m}^x(t)|g,1,0\rangle|1_m\rangle_x|0\rangle_y + \sum_m c_{5m}^y(t)|g,0,1\rangle|0\rangle_x|1_m\rangle_y + \sum_{m,n} c_{mn}^x(t)|g,0,0\rangle|1_m,1_n\rangle_x|0\rangle_y + \sum_{m,n} c_{mn}^y(t)|g,0,0\rangle|0\rangle_x|1_m,1_n\rangle_y. \tag{2}$$

The different terms in the state vector $|\psi\rangle$ represent, respectively: the dot is in the biexciton state with zero photons in the cavity; the dot is in the exciton state with one photon in the x-polarized cavity mode; the dot is in the exciton state with one photon in the y-polarized cavity mode; the dot is in ground state with two photons in x-polarized cavity mode; the dot is in the ground state with two photons in y-polarized cavity modes; and the additional possible terms due to leakage of photons from the cavity modes to the reservoirs; the suffixs to the reservoir kets represent their polarization. We note that the generation of nondegenerate two photons from a single cavity mode has been discussed in Ref. 35, when two atomic transitions in a lambda system are coupled with the same cavity mode.

By using the Schrödinger equation, applying the Weisskopf-Wigner approximation,[36–38] and introducing biexciton and exciton broadenings, we derive the following equations of motion for the probability amplitudes:

$$\dot{c}_1(t) = -ig_2^x c_2^x(t)e^{i(\omega_{ux}-\omega_c^x)t} - ig_2^y c_2^y(t)e^{i(\omega_{uy}-\omega_c^y)t} - \gamma_2 c_1(t), \tag{3}$$

$$\dot{c}_2^\alpha(t) = -ig_2^\alpha c_1(t)e^{-i(\omega_{u\alpha}-\omega_c^\alpha)t} - ig_1^\alpha\sqrt{2}c_3^\alpha(t)e^{i\Delta_c^\alpha t} - \kappa c_2^\alpha(t) - \gamma_1 c_2^\alpha(t), \tag{4}$$

$$\dot{c}_3^\alpha(t) = -ig_1^\alpha\sqrt{2}c_2^\alpha(t)e^{-i\Delta_c^\alpha t} - 2\kappa c_3^\alpha(t), \tag{5}$$

$$\dot{c}^{\alpha}_{4m}(t) = -ig^{\alpha}_1 c^{\alpha}_{5m}(t)e^{i\Delta^{\alpha}_c t} - i\Omega^*_{\alpha m}c^{\alpha}_2(t)e^{-i(\omega^{\alpha}_c-\omega_m)t} - \gamma_1 c^{\alpha}_{4m}(t),$$

$$\dot{c}^{\alpha}_{5m}(t) = -ig^{\alpha}_1 c^{\alpha}_{4m}(t)e^{-i\Delta^{\alpha}_c t} - i\Omega^*_{\alpha m}\sqrt{2}c^{\alpha}_3(t)e^{-i(\omega^{\alpha}_c-\omega_m)t} - \kappa c^{\alpha}_{5m}(t), \tag{6}$$

$$\dot{c}^{\alpha}_{mn}(t) = -i\Omega^*_{\alpha n}c^{\alpha}_{5m}(t)e^{-i(\omega^{\alpha}_c-\omega_n)t}, \tag{7}$$

where $\alpha=x$ or y, $\kappa=\pi|\Omega_{xm}|^2=\pi|\Omega_{ym}|^2$ is the spectral half width of the cavity modes (assuming uniform and equal coupling for x and y), and γ_1, γ_2 are the half widths of the exciton and biexciton levels, respectively. We note that γ_1 and γ_2 can include both radiative and nonradiative broadening, and for QDs, $\gamma_2 \approx 2\gamma_1$. We next solve Eqs. (3)–(7) to obtain c^x_{mn} and c^y_{mn}, using the Laplace transform method. The probability amplitudes for emission of a photon pair, in the long time limit, are

$$c^x_{mn}(\infty) = \frac{g^x_1\Omega^*_{xn}(\omega_m + 3\omega_n - 2\omega_x - 2\omega^x_c + 2i\kappa + 2i\gamma_1)}{(\omega_n - \omega_x + i\gamma_1)(\omega_n - \omega^x_c + i\kappa) - (g^x_1)^2} \times \frac{g^x_2\Omega^*_{xm}F_y(\omega_m,\omega_n)}{D(\omega_m,\omega_n)}, \tag{8}$$

$$c^y_{mn}(\infty) = \frac{g^y_1\Omega^*_{yn}(\omega_m + 3\omega_n - 2\omega_y - 2\omega^y_c + 2i\kappa + 2i\gamma_1)}{(\omega_n - \omega_y + i\gamma_1)(\omega_n - \omega^y_c + i\kappa) - (g^y_1)^2} \times \frac{g^y_2\Omega^*_{ym}F_x(\omega_m,\omega_n)}{D(\omega_m,\omega_n)}, \tag{9}$$

where

$$F_\alpha(\omega_m,\omega_n) = 2(g^{\alpha}_1)^2 - (\omega_m + \omega_n - \omega_\alpha - \omega^{\alpha}_c + i\kappa + i\gamma_1)(\omega_m + \omega_n - 2\omega^{\alpha}_c + 2i\kappa), \tag{10}$$

$$D(\omega_m,\omega_n) = (\omega_m + \omega_n - \omega_u + i\gamma_2)F_xF_y + (g^x_2)^2F_y(\omega_m + \omega_n - 2\omega^x_c + 2i\kappa) + (g^y_2)^2F_x(\omega_m + \omega_n - 2\omega^y_c + 2i\kappa). \tag{11}$$

The optical spectrum of the generated x-polarized photon pair is given by $S(\omega_m,\omega_n)=|c^x_{mn}(\infty)|^2$, and the spectrum for y-polarized photon pair is given by $S(\omega_m,\omega_n)=|c^y_{mn}(\infty)|^2$. The spectral functions, $S(\omega_m,\omega_n)$, represent the joint probability distribution, and thus the integration over the one frequency variable gives the spectrum at the other frequency. For example, the spectrum of the first generation of photons emitted via cavity mode is given by $S(\omega_m)=\int_{-\infty}^{\infty}S(\omega_m,\omega_n)d\omega_n$, and the spectrum of second generation of photons is $S(\omega_n)=\int_{-\infty}^{\infty}S(\omega_m,\omega_n)d\omega_m$.

From the above discussion, the state of the photon pair emitted from both the cavity modes is given by

$$|\psi\rangle = \sum_{m,n} c^x_{mn}(\infty)|1_m,1_n\rangle_x + \sum_{m,n} c^y_{mn}(\infty)|1_m,1_n\rangle_y, \tag{12}$$

where in each term the ket represents the state of the cavity mode reservoirs, and the ket suffix labels the polarization. The coefficients $c^{\alpha}_{mn}(\infty)$ are given by the analytical expressions described through Eqs. (8) and (9).

III. CAVITY-ASSISTED "ACROSS GENERATION" OF ENTANGLED PHOTONS

In the previous section, we have derived expressions for the final state of the photons generated in the biexciton-exciton cascade decay through leaky cavity modes. Depending on the coupling strength and detunings of the cavity modes from the transition frequencies in the QD, the emitted x-polarized and y-polarized photons can match in energies *within* the same generations or through *across* generations. In this section, we discuss the case when the photons match in energy in across generations. The state of the emitted photon pair is given by

$$|\psi\rangle = \sum_{k,l}[c^x_{kl}(\infty)|1_k\rangle_x|1_l\rangle_x + c^y_{kl}(\infty)|1_l\rangle_y|1_k\rangle_y], \tag{13}$$

where the first and second ket in each term show the photon of the first generation and the second generation, respectively; the second term corresponding to the y-polarized photon pair has the reverse ordering of indices compared to the first term. Although the photons of different polarizations in different generations could be degenerate in frequencies, they are distinguishable in order, namely, in time. Thus, for generating entangled photons it is necessary to make photons temporally indistinguishable as well. For erasing the temporal information, photons of the first generation are assumed to be optically delayed by time t_0. The normalized off-diagonal element of the density matrix of photons, in the polarization basis, is given by

$$\gamma = \frac{\int\int c^{x*}_{kl}(\infty)c^y_{lk}(\infty)W_{\text{opt}}(\omega_k,\omega_l)d\omega_k d\omega_l}{\int\int|c^x_{kl}(\infty)|^2 d\omega_k d\omega_l + \int\int|c^y_{kl}(\infty)|^2 d\omega_k d\omega_l}, \tag{14}$$

where $W_{\text{opt}}=\exp[-i(\omega_k-\omega_l)t_0]$ is an additional phase generated by the optical time delay. We note that the concurrence,[39] which is a quantitative measure of entanglement, for the generated state of photons $|\psi\rangle$ is equal to $2|\gamma|$; so $|\gamma|=0.5$ represents the maximum entanglement. For $t_0=0$, i.e., no time delay is employed, $W_{\text{opt}}=1$, and from Eq. (14), one gets $\gamma=0$. This shows that the phase W_{opt} is essential to erase the temporal information of photon emission from the state $|\psi\rangle$ [Eq. (13)]. For a certain value of delay t_0, the photons of the first generation and second generations can become indistinguishable and the value of $|\gamma|$ becomes maximum.

In order to better understand the results for cavity-assisted generation of entangled photons, we first consider the case when the QD is not coupled with the cavity modes. In that case, the photons are generated in the spontaneous emission through biexciton-exciton cascade decay,[20] and the coefficients c in Eq. (13) are given by

$$c^x_{kl}(\infty) = \frac{\sqrt{\gamma_2\gamma_1/2\pi^2}}{(\omega_k + \omega_l - \omega_u + i\gamma_2)(\omega_l - \omega_x + i\gamma_1)}, \tag{15}$$

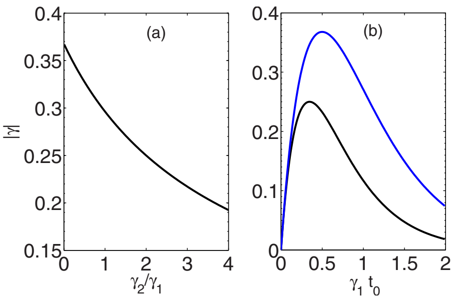

FIG. 2. (Color online) (a) Optimum value of $|\gamma|$ corresponding to a time delay $\gamma_1 t_0 = \gamma_1 \ln(1+\gamma_2/2\gamma_1)/\gamma_2$. (b) The value of $|\gamma|$ for $\gamma_2/\gamma_1 = 2$ (black) and for $\gamma_2/\gamma_1 \to 0$ (blue).

$$c_{lk}^{y}(\infty) = \frac{\sqrt{\gamma_2\gamma_1/2\pi^2}}{(\omega_k+\omega_l-\omega_u+i\gamma_2)(\omega_k-\omega_y+i\gamma_1)}. \quad (16)$$

For a QD having zero biexciton binding energy, i.e., $\omega_u = \omega_x + \omega_y$, and with a time delay t_0, from Eq. (14), one gets

$$\gamma = \frac{2\gamma_1 e^{-2\gamma_1 t_0}}{\gamma_2}(1-e^{-\gamma_2 t_0}). \quad (17)$$

From Eq. (17), we notice that γ is maximized for $\gamma_1 t_0 = \gamma_1 \ln(1+\gamma_2/2\gamma_1)/\gamma_2$. Normally for a QD, $\gamma_2/\gamma_1 \approx 2$, and the maximum value of γ is 0.25. If one can manipulate the linewidths of biexciton and excitons such that $\gamma_2/\gamma_1 \to 0$, the maximum value of $\gamma = 1/e$ can be obtained. Similar values have also been reported by simulating correlations within the density matrix formalism.[21,22] However, such manipulations of the line widths are possible in a system that includes a QD coupled with a photonic crystal cavity in the weak coupling regime.

It is important to note here, that the values of $|\gamma|$ using a time delay are quite different to the values reported by Avron *et al.*[20] The reason for this discrepancy, is that we have considered an experimentally feasible linear time delay, while the theory by Avron *et al.* is only suitable for a complex nonlinear time delay that is likely very difficult to implement in a real experiment.[40] Consequently, the maximum value of concurrence in across generation of entangled photons through linear time reordering could be 0.73, even after optimally manipulating the exciton/biexciton line widths. In Fig. 2, we show the dependence of the off-diagonal element of the photon density matrix on the value of γ_2/γ_1 [see Fig. 2(a)], and the delay time [see Fig. 2(b)].

After demonstrating in Fig. 2(a), that in the across generation of entangled photons the manipulation of the biexciton/exciton line widths is necessary for achieving higher values of entanglement, next, we show how one can manipulate the biexciton/exciton line widths by embedding a QD in a photonic crystal cavity. For QDs, γ_1 and γ_2 have radiative and nonradiative parts, and generally the nonradiative parts are larger than the radiative parts. However, in the coupled QD-photonic crystal cavity system, the radiative widths of the biexciton and excitons can be significantly larger than their nonradiative widths, and by tuning the cavity mode frequencies and couplings parameters, one can manipulate the ratio of the biexciton line width to the exciton

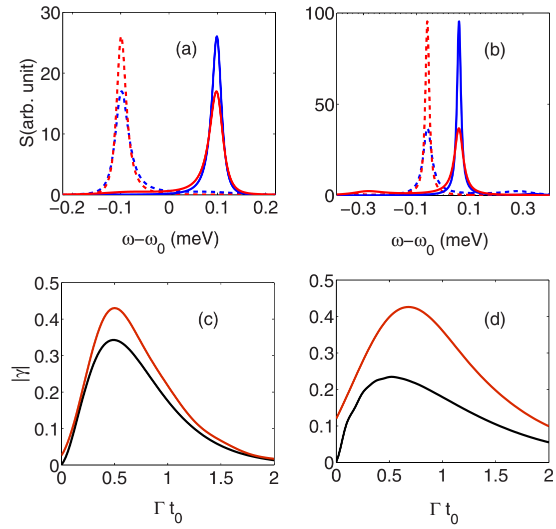

FIG. 3. (Color online) The "across generation" of entangled photons when the biexciton is also coupled with cavity modes, after reducing binding energy $\Delta_{xx} = 0$ meV. On the left, we consider weak coupling with the cavity modes degenerate with the exciton modes, and on the right we consider strong coupling with the cavity modes detuned with respect to the exciton modes. The spectrum of the photons $S(\omega)$ for $\delta_x = 0.2$ meV, $\gamma_2 = 2\gamma_1 = 0.004$ meV, $\kappa = 0.05$ meV, for (a) $g_1^x = g_2^x = g_1^y = g_2^y = 0.02$ meV, and $\Delta_c^x = \Delta_c^y = 0$ meV, and for (b) $g_1^x = g_2^x = g_1^y = g_2^y = 0.1$ meV, and $\Delta_c^x = -\Delta_c^y = -0.2$ meV. The x-polarized photons are shown in blue and the y-polarized are shown in red. For weak coupling the solid curves are for photons generated at frequency ω_x and the dotted curves are for photons generated at frequency ω_y; for strong coupling solid and dotted curves are corresponding to the transition shown in Fig. 4. In (c–d) the values of $|\gamma|$ corresponding to time delay Γt_0 are shown, where $\Gamma = g_1^{x2}\kappa/(\kappa^2 + \Delta_c^{x2})$. The red (black) curves represent results for filtered (unfiltered) photons. For (c) the filter function corresponds to two spectral windows of width $w = 0.05$ meV, centered at ω_x and ω_y, and for (d) the filter function corresponds to two spectral windows of width $w = 0.03$ meV, centered at ω_x^- and ω_y^+.

linewidth and, thus, enhance the degree of entanglement. Also, the required delay time for maximizing the entanglement can be achieved by creating path differences for photons of selected polarization and frequency. For smaller values of γ_1, one must generate a large optical path difference between photons to realize the appropriate time delays t_0, corresponding to $\gamma_1 t_0 = \gamma_1 \ln(1+\gamma_2/2\gamma_1)/\gamma_2$. However, for a QD coupled with a cavity, the linewidths of the excitons could be very large, thus, the required delay time will be significantly small and can be achieved easily in an appropriate optical delay scheme.[20]

For the *across generation* of entangled photons, we consider a QD coupled with a cavity, when the binding energy of the biexciton is suppressed to zero. We plot $|\gamma|$ for typical values of cavity couplings and detunings in Fig. 3. For the weak coupling regime, the radiative decay rates of the exciton states via the cavity modes are given by $\Gamma_1^i = g_1^{i2}\kappa/(\kappa^2 + \Delta_c^{i2})$, for $i = x, y$. The radiative decay rates for the biexciton state $|u\rangle$ into the exciton states $|x\rangle$ and $|y\rangle$ are given by $\Gamma_2^x = g_2^{x2}\kappa/[\kappa^2 + (\Delta_c^x - \delta_x)^2]$ and

$\Gamma_2^y = g_2^{y2}\kappa/[\kappa^2+(\Delta_c^y+\delta_x)^2]$. The value of $|\gamma|$ is larger when the biexciton decay rates into both exciton states are equal, i.e., $\Gamma_2^x=\Gamma_2^y$. For positive $\delta_x=\omega_x-\omega_y$, if we choose Δ_c^x negative and Δ_c^y positive, the transition $|u\rangle\to|x\rangle$ and $|u\rangle\to|y\rangle$ will be detuned from the cavity modes by $-(\Delta_c^x+\delta_x)$ and $\Delta_c^y+\delta_x$. Because of the larger detunings, the decay rates of the biexciton states become smaller than the decay rates of the excitons, which enhances the entanglement between the generated photons. In addition, the ratio Γ_2^x/Γ_1^x and Γ_2^y/Γ_1^y is maximum for cavity mode frequencies resonant with the excitons, i.e., $\Delta_c^x=\Delta_c^y=0$, and for larger values of δ_x. In Figs. 3(a) and 3(c), the cavity modes are resonant with the exciton frequencies and interact with the QD in the weak coupling regime. The maximum possible values of $|\gamma|$ are nearly 0.35 [Fig. 3(c)] which is close to the theoretical maximum value of 0.367 (specifically, $1/e$). For the strong coupling regime, the two frequencies of photons of each polarization become spectrally inseparable for small detunings. However, for larger detunings, when the photons are spectrally well resolved [see Figs. 3(b) and 3(d)], the decay rates of the biexciton to excitons and the excitons to the ground state remains nearly the same and the value of $|\gamma|$ is around 0.25.

For QDs uncoupled to cavity modes in planar photonic crystals, the radiative decay rate of the biexciton and exciton are, respectively, of the order of 0.1–0.4 μeV and 0.05–0.2 μeV.[41,42] For parameters used in Fig. 3(a), the exciton radiative decay rate is 8 μeV, which is at least 40 times enhanced, and the biexciton radiative decay rate becomes 8 times smaller than the exciton decay rate. For weak coupling, the value of $|\gamma|$ changes in the same fashion as in Fig. 2(a) by changing the ratio between of biexciton to exciton line widths. Further, in strong coupling regime, the photons are generated with the cavity field decay rate. However, in the case of strong coupling regime, one cannot manipulate the ratio between the biexciton to exciton line widths. In earlier work of cavity-assisted generation of entangled photons, the biexciton could not be coupled with cavity modes and had very long life time.[26,28] We notice also that the spectral function S is nearly one order of magnitude larger, even in weak coupling regime, than the case of when biexciton is not coupled with the cavity modes.[28] Thus, there is a clear advantage by tuning the biexciton binding energy.

To better understand the physical origin of the spectrum of Fig. 3(b), we have analytically calculated the dressed states of the biexciton and excitons in the rotating frame with frequency ω_0, neglecting all dampings, in the strong coupling regime. We relegate the details of the calculation to the appendix. For an initially excited biexciton state, the coupled cavity-QD system has five dressed states that can be expressed as the orthonormal superpositions of the bare states $|u,0,0\rangle$, $|x,1,0\rangle$, $|y,0,1\rangle$, $|g,2,0\rangle$, and $|g,0,2\rangle$. For $\Delta_c^x=-\Delta_c^y=\Delta$, $g_1^x=g_1^y=g_1$, and $g_2^x=g_2^y=g_2$, the energies of these biexciton dressed states are given by $\omega_{xx}^0=0$, $\omega_{xx}^{\pm}=\pm\sqrt{A-B}$, and $\omega_{xx}^{\pm\pm}=\pm\sqrt{A+B}$, where $A=[4g_2^2+(2\delta_x-3\Delta)^2+\Delta^2+8g_1^2]/4$, and $B=\sqrt{[2g_2^2+\Delta(2\delta_x-3\Delta)]^2+8g_1^2(2\delta_x-3\Delta)^2}/2$. After emitting the first photon via the leaky cavity mode, the system jumps to the dressed states of the excitons, which are superpositions of either $|x,0,0\rangle$ and $|g,1,0\rangle$ or $|y,0,1\rangle$ and $|g,0,1\rangle$, depending

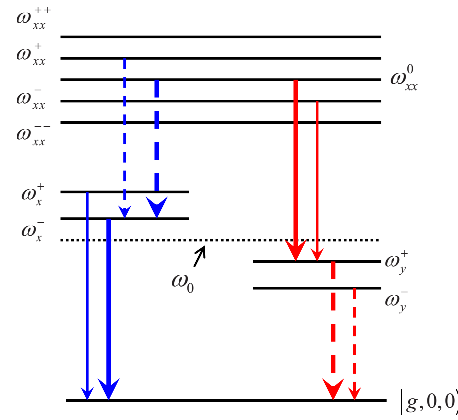

FIG. 4. (Color online) The dressed states of biexciton and exciton for *across* generation of entangled photons. The upper five states ω_{xx}^i are the dressed state of the biexciton and the lower states ω_x^i, ω_y^i are the dressed state of the $|x\rangle$ and $|y\rangle$ excitons, respectively. The bold arrows are corresponding to the dominating peaks in the emitted spectrum.

on whether the emitted photon was x-polarized or y-polarized, respectively. The frequencies of the exciton dressed states are given by $\omega_x^{\pm}=(\delta_x-\Delta\pm\sqrt{4g_1^2+\Delta^2})/2$, $\omega_y^{\pm}=(-\delta_x+\Delta\pm\sqrt{4g_1^2+\Delta^2})/2$. In principle, the first emitted photon from the dressed states of the biexciton can have *ten* peaks in the spectrum; however, for the initial state $|u,0,0\rangle$, and for off-resonant leaky cavity modes, only two peaks appear in the spectrum corresponding to the transitions $\omega_{xx}^0\to\omega_x^-$ and $\omega_{xx}^+\to\omega_x^-$ for x-polarized, and $\omega_{xx}^0\to\omega_y^+$ and $\omega_{xx}^-\to\omega_y^+$ for y-polarization; other possible transitions are negligible (see Fig. 4). Further, the peaks corresponding to the transitions $\omega_{xx}^0\to\omega_x^-$ and $\omega_{xx}^0\to\omega_y^+$ completely dominate. The second photon is emitted from the decay of the dressed states of excitons and have a two-peak spectrum corresponding to the frequencies $\omega_x^{\pm}$ or $\omega_y^{\pm}$. The peaks corresponding to frequencies ω_x^-, for the x-polarized photon, and ω_y^+, for the y-polarized photon, are largely dominating.

Although the value of $|\gamma|$ is limited by $1/e$ in the *across generation* of photons through time delay, nevertheless, the entanglement can be distilled by using a frequency filter having two narrow spectral windows of width w centered at the frequencies of degenerate peaks in the spectrum of x-polarized and y-polarized photons, say, ω_1 and ω_2. Subsequently, the response of the spectral filter can be written as a projection operator of the following form,

$$F(\omega_k,\omega_l) = \begin{cases} 1, & \text{for } |\omega_k-\omega_1| < w > |\omega_l-\omega_2|, \\ 0, & \text{otherwise}. \end{cases} \quad (18)$$

After operating on the wave function of the emitted photons [Eq. (13)] by the spectral function $F(\omega_k,\omega_l)$, we get the projected state of the filtered photon pairs. The normalized off-diagonal element of the density matrix for the filtered photons can be computed by,

$$\gamma = \frac{\int\int c_{kl}^{x*} c_{lk}^{y} W_{\rm opt}(\omega_k,\omega_l) F(\omega_k,\omega_l) d\omega_k d\omega_l}{\int\int [|c_{kl}^{x}|^2 F(\omega_k,\omega_l) + |c_{kl}^{y}|^2 F(\omega_k,\omega_l)] d\omega_k d\omega_l}. \quad (19)$$

We show in Figs. 3(c) and 3(d) (red curves) that larger values of $|\gamma|$ can be achieved by using a spectral filter. The higher values of $|\gamma|$ are achieved because of the fact that the photons along the tails in the spectrum do not get time reordered properly using a *linear* time delay, which manifests in a reduced entanglement. We find that the entanglement can be distilled by using a frequency filter with two spectral windows centered at the frequencies ω_x and ω_y for the weak coupling case and ω_x^- and ω_y^+ for the strong coupling case. Again it should be noted that the conditional probabilities after filtering, for generating entangled photon pairs, are very large [80% for Fig. 3(c) and 50% for Fig. 3(d)] because of the fact that photons are selected around the degenerate spectral peaks, and not along the degenerate tails as performed in earlier works,[8] where the conditional probabilities are much less (e.g., less than 5% conditional probabilities for 80% concurrence values). Finally, it is important to note that the effects of dephasing can also have an additional detrimental influence on entanglement of the generated state through across generation,[21] and in this regard the fast generation schemes proposed here should certainly help.

IV. CAVITY ASSISTED "WITHIN GENERATION" OF ENTANGLED PHOTONS

For *within generation* of entangled photon pairs, the x-polarized and y-polarized photons should match in frequencies within the same generations. We consider the exciton states, which have a small FSS, interact with the cavity modes in strong coupling regime so that the system forms exciton dressed states.[27] Here, we extend previous works[26,28] by considering that the biexciton state is also coupled with the same cavity modes by reducing the binding energy; however, the biexciton to excitons transitions remain more off-resonant than the excitons to ground state transitions. For generating entangled photon pairs, the dressed states of $|x\rangle$ exciton coupled with x-polarized cavity mode should match in energy with the dressed state of $|y\rangle$ exciton coupled with y-polarized cavity mode. In such a case, the x-polarized and y-polarized photon pairs generated through the decay of biexciton dressed state via exciton dressed states become indistinguishable in frequencies and thus are maximally entangled.

The state of the photon pair emitted via cavity modes can be rewritten as

$$|\psi\rangle = \sum_{k,l} [c_{kl}^{x}(\infty)|1_k\rangle_x|1_l\rangle_x + c_{kl}^{y}(\infty)|1_k\rangle_y|1_l\rangle_y], \quad (20)$$

where the coefficients c are given by the previously calculated Eqs. (8) and (9). For the state given by Eq. (20), the off-diagonal density matrix elements in the polarization basis is written as

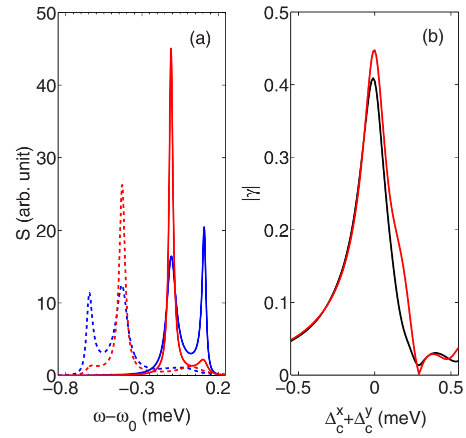

FIG. 5. (Color online) The "within generation" of entangled photons when the biexciton is also coupled with the cavity modes; the biexciton binding energy is reduced to Δ_{xx}=0.5 meV. (a) The spectrum of the photons $S(\omega)$ for δ_x=0.1 meV, $\gamma_2=2\gamma_1$=0.004 meV, κ=0.05 meV, $g_1^x=g_2^x=g_1^y=g_2^y=g$=0.1 meV, and $\Delta_c^x=-\Delta_c^y$=0.1 meV. The x-polarized photons are shown in blue and the y-polarized are shown in red; also, the solid curves are for photons generated in the decay of exciton dressed states and the dotted curves are for photons generated in the decay of biexciton dressed states. (b) The values of $|\gamma|$ for generated photons, by changing Δ_c^x for Δ_c^y=−0.1 meV. The red (black) curve represents the results for filtered (unfiltered) photons; the filter function corresponds to two spectral windows of width w=0.15 meV, centered at $\omega-\omega_0$=−0.45 meV and $\omega-\omega_0$=−0.05 meV.

$$\gamma = \frac{\int\int c_{kl}^{x*}(\infty) c_{kl}^{y}(\infty) d\omega_k d\omega_l}{\int\int |c_{kl}^{x}(\infty)|^2 d\omega_k d\omega_l + \int\int |c_{kl}^{y}(\infty)|^2 d\omega_k d\omega_l}. \quad (21)$$

We consider a positive detuning Δ_c^x and a negative detuning Δ_c^y, which are equal to the FSS, i.e., $\Delta_c^x=-\Delta_c^y=\delta_x$. In this case, the biexciton to exciton transition $|u\rangle \to |x\rangle$ and $|u\rangle \to |y\rangle$ are equally detuned by $-\Delta_{xx}$. The exciton states $|x\rangle$ and $|y\rangle$ after coupling with cavity modes form polariton states of same energies for $\Delta_c^x=-\Delta_c^y=\delta_x$.[26,28] It should be noted here that although the biexciton is more detuned, still the radiative decay rate of biexciton via cavity modes could be much larger than the radiative decay rate of biexciton uncoupled with cavity mode.

In Fig. 5(a), we show the spectrum of the photons generated in the first generation (dotted lines) and in the second generation (solid line). For efficient entanglement, it is necessary that the first generation and the second generation photons should be well resolved spectrally, therefore, a moderate $(\sim 2\sqrt{4(g_1^x)^2+\delta_x^2})$ binding energy of the biexciton is essential for the within generation scheme of entangled photons. In this case, for $\Delta_c^x=-\Delta_c^y=\delta_x$, $g_1^x=g_1^y=g_1$, $g_2^x=g_2^y=g_2$, and $\Delta_{xx} \gg g_2$, we find that the dressed states of biexciton are given by (see Appendix):

$$\omega_{xx}^0 \approx -\left(\Delta_{xx} + \frac{2g_2^2}{\Delta_{xx}}\right), \quad (22)$$

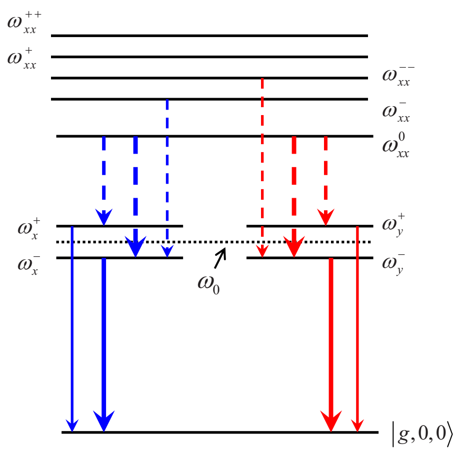

FIG. 6. (Color online) Same as in Fig. 4, but for *within generation* of entangled photons.

$$\omega_{xx}^{+} \approx \epsilon^{+} + \frac{g_2^2 \cos^2 \theta}{\Delta_{xx} + \epsilon^{+}}, \quad (23)$$

$$\omega_{xx}^{-} \approx \epsilon^{-} + \frac{g_2^2 \sin^2 \theta}{\Delta_{xx} + \epsilon^{-}}, \quad (24)$$

$$\omega_{xx}^{++} \approx - \epsilon^{-} + \frac{g_2^2 \sin^2 \theta}{\Delta_{xx} - \epsilon^{-}}, \quad (25)$$

$$\omega_{xx}^{--} \approx - \epsilon^{+} + \frac{g_2^2 \cos^2 \theta}{\Delta_{xx} - \epsilon^{+}}, \quad (26)$$

where $\epsilon_{\pm}=(-\delta_x \pm \sqrt{\delta_x^2+8g_1^2})/2$ and $\theta=\tan^{-1}[2\sqrt{2}g_1/(\delta_x+\sqrt{\delta_x^2+8g_1^2})]$. Using the parameters of Fig. 5, the dressed state of biexcitons have frequencies $\omega_{xx}^{0}=-0.54$ meV, $\omega_{xx}^{+}=0.11$ meV, $\omega_{xx}^{-}=-0.19$ meV, $\omega_{xx}^{++}=0.20$ meV, $\omega_{xx}^{--}=-0.08$ meV, and the dressed states of exciton are given by $\omega_x^{\pm}=\omega_y^{\pm}=\pm(4g_1^2+\delta_x^2)/2=\pm 0.11$. The schematic diagram of the dressed states is shown in Fig. 6. The spectra of the first-generation photons, mostly generated in the decay of biexciton dressed state ω_{xx}^{0}, have two pronounced peaks corresponding to the transitions $\omega_{xx}^{0}\rightarrow\omega_x^{\pm}$ or $\omega_{xx}^{0}\rightarrow\omega_y^{\pm}$, i.e., at -0.65 and -0.43 meV in Fig. 5. There is also a very small probability for generating photons in the transitions $\omega_{xx}^{-}\rightarrow\omega_x^{-}$ and $\omega_{xx}^{--}\rightarrow\omega_y^{-}$, corresponding to frequencies -0.08 and 0.03 meV, respectively. The spectra of the photons in the second generation have two peaks corresponding to the dressed state of excitons at ± 0.11 meV.

The calculated value of $|\gamma|$ from Eq. (21), for typical values of parameters, is shown as black curve in Fig. 5(b). For tuning the cavity mode frequencies, we fix one of the detunings Δ_c^x and Δ_c^y, and scan over the other. This type of tuning has been experimentally shown using AFM oxidization techniques,[33] and note that this scheme would be suitable to tune a large number of cavity-QD systems on the same chip. For this within generation study, we find very large values of $|\gamma|$ for the *deterministic* generation of photons. For further distilling the entanglement, spectral filters can also be used, but with a reduced probability. Using spectral filtering, the maximally entangled photons can be generated with a small reduction of probability of detection. Finally, we show the results for spectrally filtered photons in Fig. 5(b) by the red curve; the values of $|\gamma|$ are calculated using Eq. (21) after multiplying with the filter function [Eq. (18)].

V. CONCLUSIONS

In summary, we have presented quantitative theoretical models to investigate both *across generation* and *within generation* of entangle photons using single QD coupled with a photonic crystal cavity, where we have exploited the fact that the biexciton binding energy can be tuned. For zero biexciton binding energy, the concurrence for the across generation through time delay of photons is limited by $2/e$, which can be enhanced to unity using a spectral filter, at the expense of a small reduction in the probability of generation. For small biexciton binding energies, the system can be tuned to high efficiency with-in generation of entangled photon pairs. The two-photon concurrence is found to be larger than 0.8 for within generation of entangled photons, even without spectral filtering, which is likely the more practical scheme to implement experimentally.

ACKNOWLEDGMENTS

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APPENDIX: DRESSED STATES OF THE BIEXCITON

We consider the Hamiltonian for the system of the QD coupled with two modes of the photonic crystal cavity, in the rotating frame with frequency $\omega_0=(\omega_x+\omega_y)/2$, for $\Delta_c^x=-\Delta_c^y=\Delta$; with $g_1^x=g_1^y=g_1$ and $g_2^x=g_2^y=g_2$, and neglecting the coupling with environment, then

$$\frac{H_R}{\hbar} = -\Delta_{xx}|u\rangle\langle u| + \frac{\delta_x}{2}(|x\rangle\langle x| - |y\rangle\langle y|) - \left(\Delta - \frac{\delta_x}{2}\right)\hat{a}_c^{x\dagger}\hat{a}_c^x + \left(\Delta - \frac{\delta_x}{2}\right)\hat{a}_c^{y\dagger}\hat{a}_c^y + [g_1|x\rangle\langle g|\hat{a}_c^x + g_2|u\rangle\langle x|\hat{a}_c^x + g_1|y\rangle\langle g|\hat{a}_c^y + g_2|u\rangle\langle y|\hat{a}_c^y + \text{H.c.}]. \quad (A1)$$

For the across generation of entangled photons, $\Delta_{xx}=0$, we diagonalize the Hamiltonian and find the dressed energy states of the biexciton as follows:

$$\omega_{xx}^{0} = 0, \quad (A2)$$

$$\omega_{xx}^{+} = \sqrt{A - B}, \quad (A3)$$

$$\omega_{xx}^{-} = -\sqrt{A - B}, \quad (A4)$$

$$\omega_{xx}^{++} = \sqrt{A + B}, \quad (A5)$$

$$\omega_{xx}^{--} = -\sqrt{A + B}, \quad (A6)$$

with

$$A = \frac{1}{4}[4g_2^2 + (2\delta_x - 3\Delta)^2 + \Delta^2 + 8g_1^2]$$

$$B = \frac{1}{2}\sqrt{[2g_2^2 + \Delta(2\delta_x - 3\Delta)]^2 + 8g_1^2(2\delta_x - 3\Delta)^2}. \quad (A7)$$

For within generation of entangled photons, $\Delta_{xx} \neq 0$ and $\Delta = \delta_x$. From Eq. (A1) we can rewrite the Hamiltonian, H_R, in the basis of the state of the combined QD-cavity system as follows

$$H_R = -\hbar\Delta_{xx}|u,0,0\rangle\langle u,0,0| + \hbar g_2[|u,0,0\rangle\langle x,1,0| + |u,0,0\rangle\langle y,0,1| + \text{H.c.}] + H_S, \quad (A8)$$

$$H_S = -\hbar\delta_x(|g,2,0\rangle\langle g,2,0| - |g,0,2\rangle\langle g,0,2|) + \hbar g_1\sqrt{2}[|x,1,0\rangle\langle g,2,0| + |y,0,1\rangle\langle g,0,2| + \text{H.c.}]. \quad (A9)$$

After diagonalizing H_S, the eigenstates and corresponding eigenvalues of H_S are given by

$$|x_+\rangle = \cos\theta|x,1,0\rangle + \sin\theta|g,2,0\rangle, \quad \epsilon_+ \quad (A10)$$

$$|x_-\rangle = -\sin\theta|x,1,0\rangle + \cos\theta|g,2,0\rangle, \quad \epsilon_- \quad (A11)$$

$$|y_+\rangle = \sin\theta|y,0,1\rangle + \cos\theta|g,0,2\rangle, \quad -\epsilon_- \quad (A12)$$

$$|y_-\rangle = \cos\theta|y,0,1\rangle - \sin\theta|g,0,2\rangle, \quad -\epsilon_+, \quad (A13)$$

where $\epsilon_\pm = (-\delta_x \pm \sqrt{\delta_x^2 + 8g_1^2})/2$, and $\theta = \tan^{-1}[2\sqrt{2}g_1/(\delta_x + \sqrt{\delta_x^2 + 8g_1^2})]$. We can rewrite the Hamiltonian H_0 in terms of eigenstates of H_s as follows

$$H_0 = -\hbar\Delta_{xx}|u\rangle\langle u| + \hbar\epsilon^+(|x_+\rangle\langle x_+| - |y_-\rangle\langle y_-|) + \hbar\epsilon^-(|x_-\rangle\langle x_-| - |y_+\rangle\langle y_+|) + \hbar g_2\cos\theta[|u\rangle\langle x_+| + |u\rangle\langle y_-| + \text{H.c.}] - \hbar g_2\sin\theta[|u\rangle\langle x_-| - |u\rangle\langle y_+| + \text{H.c.}]. \quad (A14)$$

For $\Delta_{xx} \gg g_2$, we can use perturbation theory and obtain the eigenvalues:

$$\omega_{xx}^0 = -\Delta_{xx} - 2\Delta_{xx}\left(\frac{g_2^2\cos^2\theta}{\Delta_{xx}^2 - \epsilon^{+2}} + \frac{g_2^2\sin^2\theta}{\Delta_{xx}^2 - \epsilon^{-2}}\right), \approx -\left(\Delta_{xx} + \frac{2g_2^2}{\Delta_{xx}}\right), \quad (A15)$$

$$\omega_{xx}^+ = \epsilon^+ + \frac{g_2^2\cos^2\theta}{\Delta_{xx} + \epsilon^+}, \quad (A16)$$

$$\omega_{xx}^- = \epsilon^- + \frac{g_2^2\sin^2\theta}{\Delta_{xx} + \epsilon^-}, \quad (A17)$$

$$\omega_{xx}^{++} = -\epsilon^- + \frac{g_2^2\sin^2\theta}{\Delta_{xx} - \epsilon^-}, \quad (A18)$$

$$\omega_{xx}^{--} = -\epsilon^+ + \frac{g_2^2\cos^2\theta}{\Delta_{xx} - \epsilon^+}. \quad (A19)$$

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